Intuitionistic Subspaces in Intuitionistic Topological Spaces

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Abstract— In this paper we have used the definition and properties of intuitionistic topological spaces given in [5] to show that intersection of two intuitionistic topologies is again an intuitionistic topology but their union may not be an intuitionistic topology. Then we have defined intuitionistic accumulation point and intuitionistic derived set of an intuitionistic set. After that, we have shown that if $A \subset B$ then $A' \subset B'$, where A' and B' are intuitionistic derive sets of intuitionistic sets A and B, respectively. We have also shown that $(A \cup B)' = A' \cup B'$. The main work of this paper is the definition of intuitionistic subspace topology and discussion of some of its properties.

Keywords—Intuitionistic set, Intuitionistic topological space, Intuitionistic limit point, Relative intuitionistic topology, Intuitionistic subspace.

1 Introduction

In 1965, Zadeh[1] introduced the fuzzy set, Atanassove[2] proposed the notation of intuitionistic fuzzy set in 1983. In 1996, Coker[3] introduced the concept of intuitionistic sets and intuitionistic points. In 1997, Coker[4] defined intuitionistic fuzzy topological spaces. In 2000, Coker[5] introduced the intuitionistic topological spaces and Bayhan and Coker[6] defined T_1 and T_2 separation axioms in intuitionistic topological spaces in 2001. In 2013, Jassim[7] defined Completely normal and weak Completely normal intuitionistic topological spaces. Selvanayaki and Gnanambal[8-9] studied IGPR-continuity and homeomorphism in intuitionistic topological spaces. Kim et al.[10] and Ilango and Albinaa[11], studied some properties of intuitionistic closures and interiors in intuitionistic topological spaces. None of the papers above have discussed about intuitionistic subspaces in intuitionistic topological spaces and their properties. In this paper we have defined intuitionistic subspaces topology and discussed some of its properties.

2 Preliminaries

The definition of intuitionistic set (2.1) is given in [3] as follows:

Definition 2.1: Consider two subsets A_T and A_F of a non-empty set X such that $A_T \cap A_F = \emptyset$. Let $A = (A_T, A_F)$, then A is called an intuitionistic set (in short, IS) of X. Here A_T is called the set of members and A_F is called the set of nonmembers of A.

In fact, A_T is a subset of X agreeing or approving for a certain opinion, view, suggestion or policy and A_F is a subset of X refusing or opposing the same opinion, view, suggestion or policy, respectively.

We define $\emptyset_I = (\emptyset, X)$ and $X_I = (X, \emptyset)$, where $\emptyset_I$ is the intuitionistic empty set and X_I is the intuitionistic whole set of X. In general, $A_T \cup A_F \neq X$. The set of all ISs of X is denoted by IS(X).

The following definitions (2.2 and 2.3) are given in [3]:

Definition 2.2: For any ISs A, B and for any arbitrary collection of ISs $(A_j)_{j\in J}$ we define the following:

(i) If $A_T \subset B_T$ and $A_F \supset B_F$, then we say that $A \subset B$.
(ii) If $A \subset B$ and $B \subset A$, then we say $A = B$.
(iii) $A^c = (A_F, A_T)$ is the complement of $A = (A_T, A_F)$. Clearly A^c is also an IS.
(iv) $A \cup B = (A_T \cup B_T, A_F \cap B_F)$ is the union of A and B. Clearly $A \cup B$ is also an IS.
(v) $\cup_{j\in J} A_j = (\cup_{j\in J} A_{j,T}, \cap_{j\in J} A_{j,F})$ is the union of $(A_j)_{j\in J}$. Clearly $\cup_{j\in J} A_j$ is also an IS.
(vi) $A \cap B = (A_T \cap B_T, A_F \cup B_F)$ is the intersection of A and B. Clearly $A \cap B$ is also an IS.
(vii) $\bigcap_{j\in J} A_j = (\bigcap_{j\in J} A_{j,T}, \bigcup_{j\in J} A_{j,F})$ is the intersection of $(A_j)_{j\in J}$. Clearly $\bigcap_{j\in J} A_j$ is also an IS.
(viii) $A - B = A \cap B^c$.
(ix) $[]\, A = (A_T, A_T{}^c)$, $<>\, A = (A_F{}^c, A_F)$.

Definition 2.3: Consider an IS A of a non-empty set X and $a \in X$.

(i) $a_I = (\{a\}, \{a\}^c)$ is an intuitionistic point(IP) and $a_{IV} = (\emptyset, \{a\}^c)$ is a vanishing point of X.
(ii) If $a \in A_T$, then $a_I \in A$ and if $a \notin A_F$ then $a_{IV} \in A$.

IP(X) is the set of all intuitionistic points or intuitionistic vanishing points in X.

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3 Intuitionistic topological spaces

The definition of intuitionistic topological space (ITS), intuitionistic interior and closure (3.1 and 3.5) are given in [5] as follows:

Definition 3.1: Suppose X is a non-empty set and τ be a subclass of IS(X). Then τ is called an intuitionistic topology (in short IT) on X, if the following axioms are satisfied:

(i) $\phi_I, X_I \in \tau$,
(ii) $A, B \in \tau \Longrightarrow A \cap B \in \tau$
(iii) For each $(A_j)_{j\in J} \subset \tau$, $\cup_{j\in J} A_{j\in\tau} \in \tau$.

Then the pair (X, τ) is called ITS and each member O of τ is called an intuitionistic open set (in short, IOS) in X. If $F^c \in \tau$, for any IS F of X then it is called an intuitionistic closed set (in short, ICS) in X.

Also, $\tau_{I,0} = \{\phi_I, X_I\}$ is the intuitionistic indiscreet topology and $(X, \tau_{I,0})$ intuitionistic indiscreet topological space. $\tau_{I,1} = IS(X)$ is the intuitionistic discreet topology and $(X, \tau_{I,1})$ intuitionistic discreet topological space.

Example 3.2: Let (X, τ_0) be any ordinary topological space and let $\tau = \{(A, A^c): A \in \tau_0\}$. Then (X, τ) is an ITS

Proposition 3.3: If τ_1 and τ_2 are two intuitionistic topologies on X, then $\tau_1 \cap \tau_2$ is an intuitionistic topology on X.

Proof:
(i) Since $X_I, \phi_I \in \tau_1$ and $X_I, \phi_I \in \tau_2$ hence $X_I, \phi_I \in \tau_1 \cap \tau_2$.
(ii) Let $A_1, A_2 \in \tau_1 \cap \tau_2$. Then $A_1, A_2 \in \tau_1$ and $A_1, A_2 \in \tau_2$. But since each of τ_1 and τ_2 is an IT, $A_1 \cap A_2 \in \tau_1$ and $A_1 \cap A_2 \in \tau_2$. Accordingly, $A_1 \cap A_2 \in \tau_1 \cap \tau_2$.
(iii) Let $\{A_i: i \in I\}$ be a subclass of $\tau_1 \cap \tau_2$. Then $A_i \in \tau_1$ for each $i \in I$ and $A_i \in \tau_2$ for each $i \in I$. Since each of τ_1 and τ_2 is an IT, $\cup_i A_i \in \tau_1$ and $\cup_i A_i \in \tau_2$. Accordingly, $\cup_i A_i \in \tau_1 \cap \tau_2$.

Example 3.4: Each of the cases

$\tau_1 = \{\phi_I, X_I, (\{a\},\{b,c\}), (\{b\},\{c\}), (\phi,\{b,c\}), (\{a, b\},\{c\})\}$ and

$\tau_2 = \{\phi_I, X_I, (\{a\},\{b,c\}), (\{b\},\{c,a\}), (\{a,b\},\{c\})\}$

is an IT on $X = \{a,b,c\}$.

But, $\tau_1 \cup \tau_2 = \{\phi_I, X_I, (\{a\},\{b,c\}), (\{b\},\{c\}), (\phi,\{b,c\}), (\{a, b\},\{c\}), (\{b\},\{a\}), (\{a,b\},\phi)\}$ is not an intuitionistic topology on X. Since $(\{b\},\{c\}), (\{b\},\{a\}) \in \tau_1 \cup \tau_2$, but $(\{b\},\{c\}) \cap (\{b\},\{a\}) = (\{b\},\{a,c\}) \notin \tau_1 \cup \tau_2$.

Therefore $\tau_1 \cup \tau_2$ is not an intuitionistic topology on X.

Definition 3.5: For an ITS (X, τ) and $A \in IS(X)$, we define $Icl(A) = \cap\{G: G^c \in \tau \text{ and } A \subset G\}$ and $Iint(A) = \cup\{K: K \in \tau \text{ and } K \subset A\}$, where Icl(A) is the intuitionistic closure of A and Iint(A) is the intuitionistic interior of A.

Definition 3.6: Let (X, τ) be an ITS. An IP $p \in X$ is an intuitionistic accumulation point or limit point of an IS A of IS(X) if an only if every intuitionistic open set G containing p contains an intuitionistic point of A deferent from p. The set of all intuitionistic accumulation point of an intuitionistic set A of IS(X), denoted by A', is called the intuitionistic derived set of A.

Example 3.7: The class $\tau = \{\phi_I, X_I, (\{a\},\{b,c\}), (\{b\},\{c\}), (\phi,\{b,c\}), (\{a, b\},\{c\})\}$ be an IT on $X = \{a,b,c\}$.
Consider the IS $A = (\{b,c\},\{a\})$ of IS(X).
(i) $a \in X$ is not intuitionistic limit point of A, since the IOS $(\{a\},\{b,c\})$, which contains only IP a_I, does not contain an IP of A deferent from a_I.
(ii) $b \in X$ is not intuitionistic limit point of A, since the IOS $(\{b\},\{c\})$, which contains only IP b_I, does not contain an IP of A deferent from b_I.
(iii) $c \in X$ is intuitionistic limit point of A, since the IOS X_I, which contains IP a_I, b_I and c_I.
The intuitionistic derived set of A is $A' = (\{c\},\{a,b\})$.

Proposition 3.8: If IS A is a subset of IS B, then every intuitionistic limit point of A is also an intuitionistic limit point of B, i.e., $A \subset B$ implies $A' \subset B'$.

Solution:
Let $x_I \in A'$. Then x_I is intuitionistic limit point of A. By the definition of intuitionistic limit point, for every IOS G containing x_I and which contain at least one IP of A except x_I. Since $A \subset B$, thus for every intuitionistic open set G containing x_I and which contain at least one intuitionistic point of B except x_I. Therefore x_I is intuitionistic limit point of B, so $x_I \in B'$. Hence $A' \subset B'$.

Proposition 3.9: Let A and B be two IS of X and (X, τ) be an ITs. Then

$$(A \cup B)' = A' \cup B'.$$

Solution:
We know that if $A \subset B$ then implies $A' \subset B'$.
Since $A \subset A \cup B$ implies $A' \subset (A \cup B)'$ and $B \subset A \cup B$ implies $B' \subset (A \cup B)'$

So $A' \cup B' \subset (A \cup B)'$.

Now we have only to show that $(A \cup B)' \subset A' \cup B'$

Let $p_I \notin A' \cup B'$. Then $p_I \notin A'$ and $p_I \notin B'$. Thus there exists two IOS $G, H \in \tau$ such that $p_I \in G$ and $G \cap A \subset p_I$ and $p_I \in H$ and $H \cap B \subset p_I$. But $G \cap H \in \tau$, $p_I \in G \cap H$ and

$$(G \cap H) \cap (A \cup B) = (G \cap H \cap A) \cup (G \cap H \cap B)$$

$$\subset (G \cap A) \cup (H \cap B) \subset p_I \cup p_I = p_I$$

$\Longrightarrow (G \cap H) \cap (A \cup B) \subset p_I$.

Thus $p_I \notin (A \cup B)'$, and so $(A \cup B)' \subset A' \cup B'$.

Hence $(A \cup B)' = A' \cup B'$.

Proposition 3.10: For an ITS (X, τ) and $A \in IS(X)$. If each IP $p_I \in A$ belongs to an IOS G_p such that $G_p \subset A$. Then A is an IOS in X.

Solution:
For each IP $p_I \in A$, $p_I \in G_p \subset A$.
Hence $\cup\{G_p : p_I \in A\} \subset A$

Let $p_I \in A$ then by the given property, $p_I \in G_p$. Which implies that $p_I \in \cup\{G_p : p_I \in A\}$.

Thus $A \subset \cup\{G_p : p_I \in A\}$

Therefore $\cup\{G_p : p_I \in A\} = A$ and A is a union of intuitionistic open sets and by the definition of ITS, is IOS.

The following definition (3.11) is given in [11]:

Definition 3.11: $IS_*(X)$ denote the family of all ISs A in X such that $A_T \cup A_F = X$.

Proposition 3.12: Let $A \in IS_*(X)$ and (X, τ) be ITS. A is an ICS if and only if A contains each of its intuitionistic limit points, i.e. $A' \subset A$.

Proof:
Suppose A is an ICS and let $p_I \notin A$, i.e. $p_I \in A^c$. But A^c, the complement of an ICS, is intuitionistic open and $A \cap A^c = \phi_I$. Thus p_I is not an intuitionistic limit point of A i.e. $p_I \notin A^c$. Hence $A' \subset A$.
Again, suppose $A' \subset A$. Now we have to show that A^c is intuitionistic open.
Let $p_I \in A^c$, then $p_I \notin A$. Since $A' \subset A$, thus $p_I \notin A'$. Therefore p_I is not an intuitionistic limit point of A. By the definition of intuitionistic limit point, there exists an IOS $p_I \in G$ which does not contain any IP of A. So $p_I \in G \subset A^c$. Thus p_I is an intuitionistic interior point of A^c. Thus A^c is an IOS.

Proposition 3.13: For an ITS (X, τ) and A is a subset of X. The IS of A is $A_I = (A, \phi)$. Then the class τ^* of all intersection of A_I with τ-intuitionistic open sets of IS(X) is not an IT on A.

Solution:
let $X = \{a, b, c, d, e\}$ and let τ be the IT on X given by:
$\tau = \{\phi_I, X_I, A_1, A_2, A_3, A_4\}$, where $A_1 = (\{a, b\}, \{c, d\})$, $A_2 = (\{b, c\}, \{d, e\})$, $A_3 = (\{a, b, c\}, \{d\})$, $A_4 = (\{b\}, \{c, d, e\})$.

Let $A = \{a, b, c\}$ be a subset of X. The IS of A is $A_I = (A, \phi)$.

Observe that $A_I \cap \phi_I = (\{a, b, c\}, \phi) \cap (\phi, \{a, b, c, d, e\}) = (\phi, \{a, b, c, d, e\}) \notin IS(A)$, since $\{a, b, c, d, e\}$ is not subset of A. Hence τ^* is not an intuitionistic topological on A.

Proposition 3.14: For an ITS (X, τ) and $A \in IS(X)$. Then the class $\tau^* = \{A \cap G : G \in \tau\} \cup \{X_I\}$ is an IT on X.

Proof:
Since $\phi_I \in \tau$ and $\phi_I \cap A = \phi_I$, then $\phi_I \in \tau^*$.
Let $A_1, A_2 \in \tau^*$. Then there exists two IOS G and H such that $A_1 = G \cap A$ and $A_2 = H \cap A$. Thus $A_1 \cap A_2 = (G \cap A) \cap (H \cap A) = (G \cap H) \cap A \in \tau^*$, since $G \cap H \in \tau$.
Again let $\{A_i : i \in I\}$ be a subclass of τ^*. Then for each $i \in I$ there exists an intuitionistic τ-open set G_i such that $A_i = G_i \cap A$.

Thus $\cup_i A_i = \cup_i (G_i \cap A) = (\cup_i G_i) \cap A \in \tau^*$, since $\cup_i G_i \in \tau$.

Definition 3.15: For an ITS (X, τ) and A is a subset of X. The class $\tau_A = \{(G_T \cap A, G_F \cap A) : G = (G_T, G_F)$ and $G \in \tau\}$ is an IT on A. τ_A is called the relative IT on A or the relativization of τ to A and the ITS (A, τ_A) is called an intuitionistic topological subspace of (X, τ).

Example 3.16: let $X = \{a, b, c, d\}$ and let τ be the IT on X given by: $\tau = \{X_I, \phi_I, A_1, A_2, A_3, A_4\}$, where $A_1 = (\{a, b\}, \{d\})$, $A_2 = (\{c\}, \{b, d\})$, $A_3 = (\phi, \{b, d\})$, $A_4 = (\{a, b, c\}, \{d\})$. Again let $A = \{a, b, c\}$ be a subset of X.

Observe that
$(X \cap A, \phi \cap A) = (A, \phi) = A_I$,
$(\phi \cap A, X \cap A) = (\phi, A) = \phi_{IA}$,
$(\{a, b\} \cap A, \{d\} \cap A) = (\{a, b\}, \phi)$,
$(\{c\} \cap A, \{b, d\} \cap A) = (\{c\}, \{b\})$,
$(\phi \cap A, \{b, d\} \cap A) = (\phi, \{b\})$,
$(\{a, b, c\} \cap A, \{d\} \cap A) = (A, \phi) = A_I$
Hence, the relativization of τ to A is

$$\tau_A = \{A_I, \phi_{IA}, (\{a, b\}, \phi), (\{c\}, \{b\}), (\phi, \{b\})\}.$$

Proposition 3.17: For an ITS (X,τ) and a subset A of X, the relative IT τ_A is well-denined. In other words, $\tau_A = \{(G_T \cap A,\ G_F \cap A): G = (G_T,\ G_F) \text{ and } G \in \tau\}$ is an IT on A.

Solution:

Since τ is an IT on X, then X_I and ϕ_I belongs to τ.
Hence $(X \cap A,\ \phi \cap A) = (A,\ \phi) = A_I \in \tau_A$ and $(\phi \cap A,\ X \cap A) = (\phi,\ A) = \phi_{IA} \in \tau_A$.
Let $H_1, H_2 \in \tau_A$. Then there exist $G_1, G_2 \in \tau$ such that
$H_1 = (G_{1T} \cap A,\ G_{1F} \cap A)$, where $G_1 = (G_{1T}, G_{1F})$ and
$H_2 = (G_{2T} \cap A,\ G_{2F} \cap A)$, where $G_2 = (G_{2T}, G_{2F})$.
But $G_1 \cap G_2 = (G_{1T} \cap G_{2T}, G_{1F} \cup G_{2F}) \in \tau$, since $G_1, G_2 \in \tau$.

$$\text{Hence } H_1 \cap H_2 = \big((G_{1T} \cap A) \cap (G_{2T} \cap A),\ (G_{1F} \cap A) \cup (G_{2F} \cap A)\big) = \big((G_{1T} \cap G_{2T}) \cap A,\ (G_{1F} \cup G_{2F}) \cap A\big) \in \tau_A,$$

since $G_1 \cap G_2 \in \tau$.
Again let $\{H_i : i \in I\}$ be a subclass of τ_A. By the property of τ_A, for each $i \in I$ there exist $G_i \in \tau$ such that
$H_i = (G_{iT} \cap A,\ G_{iF} \cap A)$, where $G_i = (G_{iT}, G_{iF})$
But $\cup_i G_i \in \tau$, since $G_i \in \tau$ for each $i \in I$.

Now,

$$\cup_i H_i = \cup_i (G_{iT} \cap A, G_{iF} \cap A) = \big(\cup_i (G_{iT} \cap A),\ \cap_i (G_{iF} \cap A)\big) = \big((\cup_i G_{iT}) \cap A,\ (\cap_i G_{iF}) \cap A\big) \in \tau_A, \quad \text{since } \cup_i G_i \in \tau.$$

Therefore τ_A is an intuitionistic topology on A.

Proposition 3.18: Every intuitionistic subspace of an intuitionistic discrete topological space is discrete.

Solution:

Let (A,τ_A) be an intuitionistic subspace of an intuitionistic discrete topological space (X,τ). Let G be an IS of A, where $G = (G_T,\ G_F)$. Since $G_T \subset A, G_F \subset A$ and $A \subset X$, then $G_T \subset X, G_F \subset X$. That is $G = (G_T,\ G_F)$ is an IS of X. Again, since (X,τ) an intuitionistic discrete topological space, then $G = (G_T,\ G_F)$ is an intuitionistic τ-open set.
Observe that, $(G_T \cap A,\ G_F \cap A) \in \tau_A$, since (A,τ_A) is an intuitionistic subspace of (X,τ). Which implies that $(G_T,\ G_F) \in \tau_A$, since $G_T \cap A = G_T$ and $G_F \cap A = G_F$. Therefore, $G \in \tau_A$, i.e. every IS of A is an intuitionistic τ_A-open set. Hence (A,τ_A) is an intuitionistic discrete topological space.

Proposition 3.19: Every intuitionistic subspace of an intuitionistic indiscrete topological space is indiscrete.

Solution:

Let (A,τ_A) be an intuitionistic subspace of an intuitionistic indiscrete topological space (X,τ). Let $G = (G_T,\ G_F)$ be any intuitionistic τ_A-open set. By the property, either $G = (G_T,\ G_F) = (X \cap A,\ \phi \cap A) = (A, \phi) = A_I$ or $G = (G_T,\ G_F) = (\phi \cap A,\ X \cap A) = (\phi, A) = \phi_{IA}$, since (X,τ) an intuitionistic indiscrete topological space and (A,τ_A) be an intuitionistic subspace of (X,τ). Thus every member of τ_A is either A_I or ϕ_{IA}. Hence (A,τ_A) is an intuitionistic indiscrete topological space.

Proposition 3.20: Let (X,τ) be an intuitionistic subspaces of (Y,τ^*) and let (Y,τ^*) be an intuitionistic subspaces of (Z,τ^{**}). Then (X,τ) is also an intuitionistic subspaces of (Z,τ^{**}).

Solution:

Since $X \subset Y \subset Z$, (X,τ) is an intuitionistic subspaces of (Z,τ^{**}) if and only if $\tau_X^{\ **} = \tau$. Let $G \in \tau$ and (X,τ) be an intuitionistic subspaces of (Y,τ^*), then $\exists\ G^* \in \tau^*$ such that $G = (G_T^{\ *} \cap X,\ G_F^{\ *} \cap X)$, where $G^* = (G_T^{\ *},\ G_F^{\ *})$.

Again, since $G^* \in \tau^*$ and (Y,τ^*) be an intuitionistic subspaces of (Z,τ^{**}), then $\exists\ G^{**} \in \tau^{**}$ such that $G^* = (G_T^{\ **} \cap Y,\ G_F^{\ **} \cap Y)$, where $G^{**} = (G_T^{\ **},\ G_F^{\ **})$.

Thus $G_T^{\ *} = G_T^{\ **} \cap Y$ and $G_F^{\ *} = G_F^{\ **} \cap Y$,

$$\begin{aligned} \text{so } G &= \big((G_T^{\ **} \cap Y) \cap X,\ (G_F^{\ **} \cap Y) \cap X\big) \\ &= \big(G_T^{\ **} \cap (Y \cap X),\ G_F^{\ **} \cap (Y \cap X)\big) \\ &= (G_T^{\ **} \cap X,\ G_F^{\ **} \cap X), \text{ since } X \subset Y, \text{ thus } Y \cap X = X. \end{aligned}$$

Hence $G \in \tau_X^{\ **}$. Accordingly, $\tau \subset \tau_X^{\ **}$.
Now suppose, $G \in \tau_X^{\ **}$, i.e. $\exists\ H \in \tau^{**}$ such that $G = (H_T \cap X,\ H_F \cap X)$, where $H = (H_T,\ H_F)$. Since (Y,τ^*) be an intuitionistic subspaces of (Z,τ^{**}) and $H \in \tau^{**}$, then $(H_T \cap Y,\ H_F \cap Y) \in \tau_Y^{\ **} = \tau^*$. Again since (X,τ) be a subspaces of (Y,τ^*) and $(H_T \cap Y,\ H_F \cap Y) \in \tau^*$.
Then $\big((H_T \cap Y) \cap X,\ (H_F \cap Y) \cap X\big) \in \tau_X^{\ *} = \tau$.
Which implies that $\big(H_T \cap (Y \cap X),\ H_F \cap (Y \cap X)\big) \in \tau$, since $X \subset Y$, thus $Y \cap X = X$. Therefore, $(H_T \cap X,\ H_F \cap X) \in \tau$ and we have $G \in \tau$. Accordingly, $\tau_X^{\ **} \subset \tau$.

Proposition 3.21: Let (Y,τ_Y) be an intuitionistic subspaces of (X,τ). If $E = (F_T \cap Y,\ F_F \cap Y)$, where $F = (F_T,\ F_F)$ is intuitionistic τ-closed set in IS(X) then $E \in IS(Y)$ is intuitionistic τ_Y-closed set.

Solution:

Let $F = (F_T,\ F_F)$ is intuitionistic τ-closed set in IS(X), then $F^c = (F_F,\ F_T)$ is an intuitionistic τ-open set. Since (Y,τ_Y) be an intuitionistic subspaces of (X,τ), then by the given property $(F_F \cap Y,\ F_T \cap Y)$ is an intuitionistic τ_Y-open set in IS(Y). Therefore, $(F_T \cap Y,\ F_F \cap Y)$ is an intuitionistic τ_Y-closed set in IS(Y). By the given property $E = (F_T \cap Y,\ F_F \cap Y)$, where $F = (F_T,\ F_F)$. Hence E is an intuitionistic τ_Y-closed set in IS(Y).

CONCLUSION

In this paper our results are Proposition 3.8-3.10 and 3.12-3.14, Definition 3.15, Example 3.16 and Proposition 3.17-3.21. The main work of this paper is the definition of intuitionistic subspace topology (Definition 3.15) and some of its properties. We hope that these results will be helpful for further studies of intuitionistic topological space.

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ETHICAL STATEMENT

We have not intentionally engaged in or participate in any form of malicious harm to another person or animal. We have no conflict of interest.

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